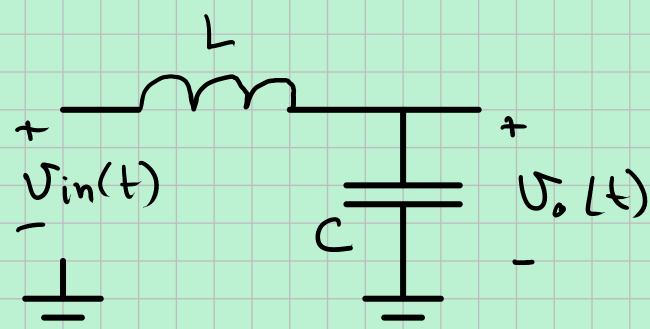
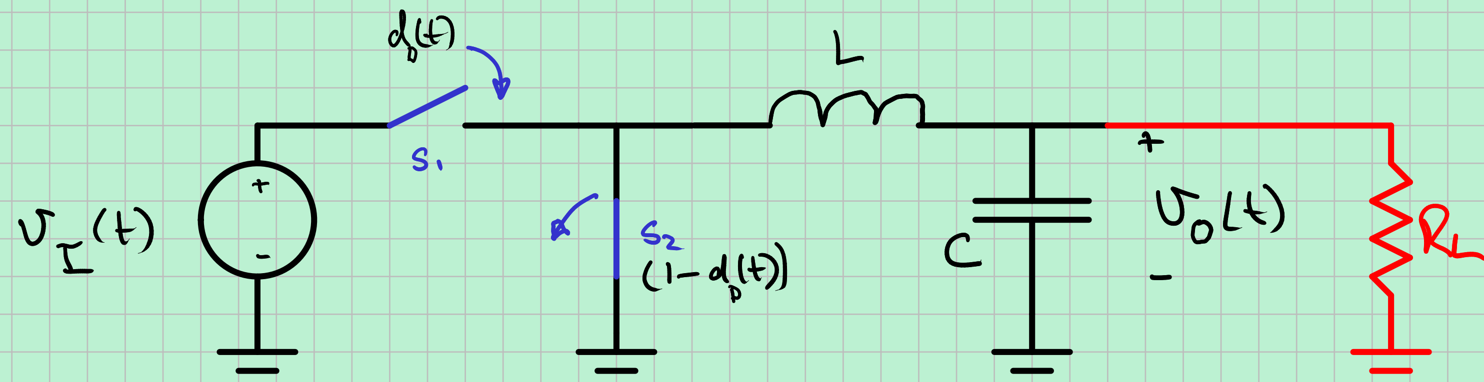

L
+
$v_{in}(t)$
−
C
+
$v_o(t)$
−



Effect of R_L load resistor in red.

$$L\frac{di_L}{dt} + v_c(t) = v_I$$

$$\dot{i}_L = -\frac{1}{L}v_c + \frac{v_I}{L}$$

$$i_L = C\frac{dv_c}{dt} = C\dot{v}_c + \frac{v_c}{R_L}$$

$$\dot{v}_c = \frac{1}{C}i_L - \frac{v_c}{R_L C}$$

$$\begin{bmatrix}\dot{i}_L \\ \dot{v}_c\end{bmatrix} = \underbrace{\begin{bmatrix}0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{R_L C}\end{bmatrix}}_{\underset{\sim}{A}}\underbrace{\begin{bmatrix}i_L \\ v_c\end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix}\frac{1}{L} \\ 0\end{bmatrix}}_{\underline{B}} v_I$$

when S_1 closed ($d(t)$ of the time)

$$\begin{bmatrix}\dot{i}_L \\ \dot{v}_c\end{bmatrix} = \begin{bmatrix}0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{R_L C}\end{bmatrix}\begin{bmatrix}i_L \\ v_c\end{bmatrix} + 0$$

when S_2 is closed ($(1-d(t))$ of the time)

Averaging:

$$\dot{x} = \underset{\sim}{A}\,\underline{x} + \underline{B}\cdot d_D(t)\, v_I(t)$$

Perturbation Method:

$$d_D(t) = D_D + d_d(t) \quad \& \quad v_I(t) = V_I + v_i(t)$$

$$d_D(t)\, v_I(t) = (D_D + d_d(t))(V_I + v_i(t))$$

$$\cong D_D V_I + V_I d_d(t) + D_D v_i(t) + \cancel{d_d(t) v_i(t)}^{\approx 0}$$

$$x_s(t) = \underline{X_s} + \underline{x_s}(t)$$

$$\underline{\dot{x}_s}(t) = \cancel{\frac{d\underline{X_s}}{dt}}^{0} + \underline{\dot{x}_s}(t)$$

$$\underline{\dot{x}_s}(t) = \underset{\sim}{A}\left(\underline{X_s} + \underline{x_s}(t)\right) + \underline{B}\left(D V_I + V_I d_d(t) + D_D v_i(t)\right)$$

$$\dot{x}_s(t) = \underset{\sim}{A}\underline{X_s} + \underline{B} D V_I + \underset{\sim}{A} x_s(t) + \underline{B}\,[V_I, D_D]\begin{bmatrix} d_d(t) \\ v_i(t) \end{bmatrix}$$

setting small signals to zero:

$$\underset{\sim}{A}\underline{X_s} + \underline{B} D_D V_I = 0$$

$$\underline{X_s} = -\underset{\sim}{A}^{-1}\underline{B} D_D V_I \qquad \text{This is the Q point}$$

$$\underline{\dot{x}_s}(t) = \underset{\sim}{A}\,\underline{x_s}(t) + \underbrace{\begin{bmatrix} 1/L \\ 0 \end{bmatrix}\begin{bmatrix} V_I & D_D \end{bmatrix}}_{\begin{bmatrix} V_I/L & D_D \\ 0 & 0 \end{bmatrix}}\begin{bmatrix} d_d(t) \\ v_i(t) \end{bmatrix}$$

This last is the small signal model.

Let's find the Q point:

$$\underset{\sim}{A}^{-1} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & \frac{1}{R_L C} \end{bmatrix}^{-1} = LC\begin{bmatrix} \frac{1}{R_L C} & \frac{1}{L} \\ -\frac{1}{C} & 0 \end{bmatrix} = \begin{bmatrix} \frac{L}{R_L} & C \\ -L & 0 \end{bmatrix}$$

$$\begin{bmatrix} I_L \\ V_C \end{bmatrix} = \underline{X_s} = -\underset{\sim}{A}^{-1}\underline{B} D_D V_I = \begin{bmatrix} \frac{L}{R_L} & -C \\ L & 0 \end{bmatrix}\begin{bmatrix} 1/L \\ 0 \end{bmatrix} D_D V_I = \begin{bmatrix} \frac{D V_I}{R_L} \\ D_D V_I \end{bmatrix}$$